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# NEWTON'S SECOND LAW OF MOTION, F=MA; EULER'S OR NEWTON'S?

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## ABSTRACT

**Objective:** F =ma is taught as Newton's second law of motion all over the world. But it was given by Euler in 1775, forty-eight years after the death of Newton. It is debated here with scientific logic. **Methods/Statistical analysis**: The discussion partially deals with history of science so various aspects are quoted from original references. Newton did not give any equation in *the Principia* for second, third laws motion and law of gravitation. Conceptually, in Newton's time, neither acceleration nor second derivative was employed, so it was impossible for him to write F =ma. Descartes laws preceded Newton's laws; these aspects need to be taken in account. Newton separated physics from natural science. **Findings:** The Principia's second law at pages 19-20 states as– *the alteration in motion is proportional to force* i.e. $F = (v - u)$. In the existing literature, motion is regarded as 'quantity of motion' (amount or quantum or magnitude of motion). Furthermore, 'alteration', i.e. difference, is regarded as 'rate of change of '. These are not scientifically justified. F = (v-u) is not taught and mentioned anywhere, however Euler's F=ma is regarded as Newton's second law of motion. If Proposition 1, Proposition VI and Proposition LXVI are critically analyzed, then $F = ma$ does not follow from them in any way. Before Newton' time , tradition of representing change in velocity by length of segment is never justified. F=ma can never be derived from circular motion as it describes linear motion. More over laws are based on scientific logic not on traditions. Further law of gravitation was given in nearly 12 propositions and was not pointed as product of masses of bodies. The concept of units and dimensions was initiated by Fourier in 1822 i.e. 136 years after publication of the principia.
**Application/Improvements:** The credit of discovery of F =ma must not be given to Newton. Newton started beginning of physics by defining new terms, but the beginning must not be regarded end.

**KEYWORDS**: Newton laws, F = (v-u), Euler, F =ma, Descartes laws

## I. DEVELOPMENT OF LAWS OF MOTION

Aristotle (384BC-322BC) believed in doctrine of effect and cause. Everything that is in motion must be moved by something. A table is pushed or pulled on the floor then it moves as long as external force acts on it. The table stops when external force ceases to act. Aristotle stated that rest (velocity =0) is natural state of body, and it starts moving when by external force acts on it. Thus heavier the body more force is needed to push or pull it, which is justified in daily life observations. Thus scientists find Aristotle's early work very significant and even quoted[1] that 'without Aristotle's *Physics* there would have been no Galileo'.

Galileo perceived experiments in hypothetical system when all resistive forces are eliminated from the system, then body once set in motion always remains in state of uniform velocity (constant velocity). Thus Galileo on the basis of such experiments perceived that a moving body maintains its constant speed in straight line unless no external force acts on it. Thus Galileo put forth that movement of body with constant velocity is natural tendency of body, and it stops due to resistive forces[2] Galileo's law of inertia is given by *"A body moving on a level surface will continue to move in the same direction at a constant speed unless disturbed."*

### 1.1 Descartes Laws of Motion

Thus Aristotle's assertion (natural tendency of body is to remain at rest) and Galileo's law of inertia ( natural tendency of body is to remain in state of uniform motion ) are opposite to each other. Both utilize the fact that no external force acts on body. Renne Descartes, in 1644 in his book **Principles of Philosophy** elaborated Galileo's law of inertia in first two laws of motion[3-4] . Further Descartes third law of motion explains the collision of bodies of moving bodies; it is independent of Galileo's law of inertia or Aristotle's assertion. But Newton's third law of motion has resemblance with this law.

Law 1

*Each thing, in so far as it is simple and undivided, always remains in the same state, as far as it can, and never changes except as a result of external causes. ... Hence we must conclude that what is in motion always, so far as it can, continues to move.* (Principles Part II, article. 37)

Law 2

*Every piece of matter, considered in itself, always tends to continue moving, not in any oblique path but only in a straight line*. (Principles Part II, article. 39)

Law 3

*When a moving body collides with another, if its power of continuing in a straight line is less than the resistance of the other body, it is deflected so that, while the quantity of motion is retained, the direction is altered; but if its power of continuing is greater than the resistance of the other body, it carries that body along with it, and loses a quantity of motion equal to that which it imparts to the other body.* (Principles Part II, article. 40)

It is evident that Descartes' first two laws are just other form of Galileo's law of inertia. Descartes third law of motion is independent of Galileo's Laws of inertia. Thus it is Descartes original work and preceded Newton's third law of motion. Neither Galileo nor Descarte gave any hint to find out the magnitude of force required for change the constant velocity of body.

Equivalently, Newton gave new three laws of motion after 42 years of publication of *Principles of Philosophy.* Newton wrote the famous masterpiece *The Mathematical Principles of Natural Philosophy* in 1686; and did not change the laws for next forty years i.e. in second edition 1713 and third edition 1726 (published a year before Newton's death).

**Newton's First Law of motion:** Further first part of Newton's first law of motion also states, the body remains in state of rest (v=0) when no external force (F=0) acts on it , which is Aristotle's assertion. Thus Newton's law also assumes additional validity of Aristotle's assertion. In second part Newton's law assumes validity of Galileo's laws of inertia i.e. body maintains its constant velocity in absence of impressed force (F=0). Thus Newton's first law of motion is fusion or quotation of two opposite tendencies or perceptions already existing literature right since days of Galileo and Descarte. Thus Newton's first law of motion is re-quotation of two independent laws (Aristotle assertion and Galileo's law of inertia) form existing literature into one.

### 1.2 Newton's Second law of motion

Galileo's law of inertia and Descartes two laws of motion imply that bodies remain in state of uniform motion in straight line when no resistive forces act on the body. Both the doctrines do not point out the calculations of force acting on body when uniform velocity of body changes. The reason may be that they deal with constant velocity. However in the second law of motion Newton gave the equation to find out the force when velocity changes. The reason may be that law of inertia and Descartes second law deals with the case when body moves with constant velocity. An equation may be speculated that for a system when velocity varies number of times in the interval due to variation of the resistive forces or other factors; as science is not static body.

Newton's second law of motion points out the magnitude of force when velocity of body changes i.e. alternation in force is proportional to impressed force. Mathematically, Force =change in velocity. It is discussed that now second law of motion is based on definition of equation of force, F =ma which was given by Swiss mathematician Leonhard Euler. The Principia's second law of motion is not used at all in physics. However Euler's law ( F=ma) is used as Newton's second law of motion.

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***Table I Aristotle, Galileo, Descartes and Newton on first two laws of motion***

| Sr. No. | Scientist | First Law | Second Law | Comment |
|---|---|---|---|---|
| 1 | Newton 1686 | i) Body remains at rest Impressed force F =0<br>ii) Body moves with uniform velocity , if resistive force f =0 | F = (v-u) | Practical system resistive forces present.<br>Hypothetical System No resistive force |
| 2 | Galileo 1609 | i) -------<br>ii) Body moves with uniform velocity if resistive forces f =0 | --------<br>F=0 | -------<br>Hypothetical System No resistive force acts |
| 3 | Descartes 1644 | Same as Galileo | Same as Galileo | Same as Galileo |
| 4 | Aristotle (384BC-322BC) | i) Body remains at rest Impressed force F =0<br>ii) ------------- | ----- | Practical system resistive forces present |

1.3 **Critical discussion on Newton's and Descartes Third law of motion**.

**Newton's Third Law of Motion:** Newton's third law of motion implies that as equality between action and reaction in opposite direction universally i.e. for all bodies and at all places. Further Newton described the law for both cases when after interactions bodies remain at rest or move.

(a) Newton gave first two examples in *the Principia* at page 20, when body remains at rest e.g. when finger pushes the stone, stone is also pushed by the finger. Secondly, when a horse pulls the stone tied to it , the stone also pulls the horse. Thus action and reaction are equal and opposite. In both cases stone remains at rest, work done by each system is zero. The other example in daily life is book lies on the table. Such and similar other examples are strongly given in favor of Newton's third law of motion.

(b) Whereas in third example, Newton considered the case when moving body collides the other body causing it to move.

*If a body impinges upon another and by its force change the motion of the other, that body also (because of the quality of, the mutual pressure) will undergo an equal change, in its own motion, towards the contrary part.*

Thus if body A changes the motion (velocity) of body B by colliding, the motion (velocity) of body A is also changed by the same amount as that of body B but in opposite direction. Newton just stated this but did not elaborate the same. The equations of collisions for projectile and target before and after collisions have been written in the existing literature. The basis of collision is the same as originated from Newton's third law of motion (Descartes third law), so both must be interpreted simultaneously[5] in view of law. In one dimensional elastic collisions the equations for initial and final velocities of projectile are target are calculated in existing literature. These are required to be interpreted in terms of third law of motion. These equations were written when laws of conservation of momentum and energy were defined. Further equation for momentum, kinetic energy can be expressed in terms velocity. Newton had given reason for change in motion (velocity) as quality of the mutual pressure, but in spite of this fact changes in projectile are same as in case of target but in opposite direction i.e. action and reaction are equal and opposite universally.

**Descartes Third Law of Motion**: Basically, in Newton's and Descartes laws, the language is different but theme may be the same. Newton did not discuss the third example in *the Principia*, with details as in case of Descartes law.

In this case bodies move and work is done by the system. In simple words , 'the power of continuing' may be regarded as 'force' exerted by projectile and , the 'resistance of other body' implies force (resistive in nature) which is exerted by target on the projectile. In Newton's compact terminology, these are action (the power of

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continuing) and reaction (resistive force of body); both are equal and opposite. Now a day the nomenclature or terminology may be further different.

Descartes law can be further extended for the third possibility e.g. *if its power of continuing in a straight line is equal the resistance of the other body*. Then the system remains at rest and no work is done. Descartes had not considered this case. This case is similar to Newton's first two examples when body remains at rest, and work done is zero. Thus Newton's third law and Descartes third law (preceded Newton's law) appear to give similar results if critically analyzed.

The dominance of 'power of continuing of projectile' and 'resistance of other body' i.e. target, obviously depends upon inherent characteristics of bodies. Such aspects can be better understood in Descartes third law of motion, than Newton's third law as former is more elaborative. Author has already discussed in such and similar interactions[5] and concluded that the inherent characteristics, nature, compositions, flexibility, rigidity, magnitude, size, elasticity, shape , distinctiveness of interacting bodies, mode of interactions, point of impact etc. play significant roles. The bodies affect the results e.g. bodies may be of steel, wood, rubber, cloth, wool, sponge, spring, typical plastic, porous material, air / fluid filled artifact, mud or kneaded flour or chewing gum specifically fabricated material etc. This discussion of such factors with details has lead to generalization of third law of motion[5] .Thus understanding of scientific laws is continuous process.
In The World (also called ***Treatise on the Light*** ) published 1664, he states[6]:

*"the virtue or power in a body to move itself can well pass wholly or partially to another body and thus no longer be in the first; but it cannot no longer exist in the world" (AT XI 15).*
This statement also relates to third law of motion. This is one of the first formulations of the conservation principle and the concept of 'force'.

***Table II Comparison of Descartes and Newton's Third Law of Motion***

| Sr. No | Scientist | Third Law of Motion | | | Generalization |
|---|---|---|---|---|---|
| 1 | | Terms | Description | Effect | |
| | Descarte (1644) | Power of continuing (P) & Resistance by body (R) | P<R | Projectile is deflected | Yes |
| | | | P> R | Projectile moves ahead | |
| | | | P= R | Projectile remains at rest | |
| | | Characteristics of body | | Play significant role but not studied. | |
| 2 | Newton (1686) | Action & | Action = Reaction | Always equal universally | |

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| | | | |
|---|---|---|---|
| | | Reaction | Yes |
| | | Characteristics of body      Theoretically No role | |

**2.0 The Principia's and prevalent or prevailing second law of motion.**

The prevalent form of Newton's second law of motion is,
*The rate of change of momentum is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*'

$F \propto ma$ or $F = Kma$ (1)

The value of coefficient of proportionality K, is determined experimentally like other coefficients e.g. coefficient of viscosity , coefficient of thermal conductivity etc. But it is regarded as unity in all cases i.e. universally. Thus,

$F = ma$ (2)

Newton stated his second law of motion in *the Principia*[7] as follows:
"*The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*"

As already mentioned, Newton did not write any equation for his laws of motion. To understand we can write,

$F \propto (v-u)$ or $F = k(v - u)$ (3)

where in $k$ is coefficient of proportionality . In existing physics all coefficients are determined experimentally. However k is regarded as unity analogous to K.

Thus in view of this both eqs.(1,3) become

$F = (v-u) = dv$ (4)

$F = ma$ (2)

Euler too have given this equation independently[8-11] .This definition directly follows from Euler's equation of force[8-9] i.e. $F = ma = \dfrac{dp}{dt}$

The LHS of eq.(2) and eq.(4), represent force in LHS, both the definitions are expressed in proportionality form , and also phrase 'and is made in the direction of the right line in which that force is impressed' is common , thus both definitions appears to have similarity. However both are entirely different conceptually and scientifically as the terms or phrases 'rate of change of momentum' and 'alternation in motion are scientifically different.

This definition and equation were never given by Newton. The definitions are entirely different, and can be clearly understood from the meanings of 'motion' and `quantity of motion', 'alteration' and 'rate of change of ' are entirely different.

(i) **Absolute Motion (motus absolutus)**

After the definitions in scholium Newton wrote, "I do not define time, space, place and motion, as being well known to all." But Newton should have restated or explained the meanings of Time, Space, Place and Motion in a paragraph in *the Principia* which consists of three Books of over 570 pages. At that time there was no tradition or conceptual basis to represent laws or concepts in terms of mathematical equations. The concepts and equations were written afterwards, and now we have reached age of computer programming and robotics.

Further Newton did not mention acceleration neither in newly defined quantities nor in previously known quantities in *the Principia*. It is clearly meant that acceleration was unknown in Newton's time. Also he wrote three editions in 1686, 1713 and 1726, Newton did not change the laws of motion. Thus Newton did not define motion separately as it was too elementary and understood in terms of velocity earlier also i.e. since days of Aristotle, Galileo etc. Aristotle, Galileo and other scientists interpreted motion in terms of velocity frequently.

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Newton[7] illustrated motion as absolute motion in Scholium of DEF. VIII, page 10. Absolute motion is the translation of a body from one absolute place into another; and relative motion, the translation from one relative place into another. In physics translation is defined as movement of a body from one point to other, i.e. a body possesses velocity. While units and dimensions were defined then velocity was used in to express the movement of body. In explanation Newton quoted the example of ship, considering motion as velocity.

*"* As if that part of the earth, where the ship is, was truly moved toward the east, with a velocity of 10010 parts; while the ship itself, with a fresh gale, and full sails, is carried towards the west, with a velocity expressed by 10 of those parts; but a sailor walks in the ship towards the east, with 1 part of the said velocity; then the sailor will be moved truly in immovable space towards the east, with a velocity of 10001 parts, and relatively on the earth towards the west, with a velocity of 9 of those parts."
Thus in Newton's time there were no units of velocity like m/s and dimensions $M^0LT^{-1}$ hence he quoted velocity of 10010 parts, rather than velocity 10010 m/s. Further it implies that Newton expressed motion in terms of velocity.

Newton stated first law of motion in terms of uniform motion which is uniform velocity. Earlier Galileo stated the law of inertia in similar other way. Further for uniform motion or in non-uniform motion, we draw a graph between velocity-time graph, not motion-time graph. Thus motion is represented by velocity. The various definitions[7] in *the Principia* imply motion as velocity.
First Law of Motion: *Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.*
**Definition IV:** *In impressed force is an action exerted upon a body in order to change its state, either of rest or of moving uniformly forward in a right line .*
Thus moving uniformly forward means moving with uniform velocity.
**Definition III :** *The Vis Insita or Innate Force of Matter , is power of resisting by which everybody , as much as in it lies , endeavours to preserve it in present state , whether it be of rest or of moving uniformly moving forward in a right line.*

Again moving uniformly forward means moving with uniform velocity.
Galileo practically illustrated the law of inertia in terms of velocity. Also Aristotle expressed motion in terms of speed. Later on concept of units and dimensions was established, and velocity was regarded as physical quantity (units m/s, dimensions $M^0LT^{-1}$). The velocity possesses units and dimensions and is a vector quantity. Thus when we mean body is in motion it implies that it possesses velocity, and if body is at rest then it does not possess velocity. The motion neither has units and dimensions not it is characterized as vector or scalar. But now a days velocity does posses units (m/s) and dimensions ($M^0LT^{-1}$ ) and is vector not scalar. Thus due to lack of technological development, the old name for velocity was motion.
The dimensional analysis was initiated by Fourier[12-13] in 1822, so this test was not applicable to equations. The body is said to be in motion when it moves or possesses velocity e.g. a bus in motion. But quantitative information is provided by velocity as bus moves with velocity 40km/hr from east to west. Thus in any equation LHS and RHS are not expressed in terms of motion, but equation is expressed in terms of velocity.

Further meaning of motion in oxford dictionary is *The action or process of moving or being moved* e.g. *planetary motion.* Now more specifically we express the motion regarding it as orbital velocity or escape velocity and their values are specifically calculated. For example orbital velocity of the earth is 29.75 km/s and escape velocity 11.2 km/s. Whereas the orbital velocity for Jupiter is 13.6 km/s and escape velocity is 59.5km/s . Thus now velocity is specifically used in description as it is physical quantity having units and dimensions. In physics motion has no units and dimensions, hence it cannot be said to be physical quantity.

Thus no equation is written in term of motion but in terms of velocity e.g. $v = u+at$ , $v^2 - u^2 = 2aS$ , $V=HS$ (Hubble's law). It is pertinent to mention here that third law of motion is expressed in terms of 'action' and 'reaction', which too do not possess units and dimensions. So in Newton's time the phenomena were explained philosophically and qualitatively. Newton started initiation of the physics in systematic way, which is in the state of development even now. Earlier physics was part of natural philosophy (the philosophical study of nature and the physical universe that was dominant before the development of modern science). So Newton initiated beginning of physics, which should not be regarded as end.

Like any other branch of knowledge, physics was gradually developed. Also the unit of force the dyne[14] was initially defined in 1861 about 184 years after publication of the first edition of *the Principia*. But this definition was unacceptable to the Committee of the British Association for the Advancement of Science[15] The 9th Confe´rence Ge´ne´rale des Poids et Mesures held in 1948, then adopted the name "Newton" for the unit of force[16] in resolution7

## II. QUANTITY OF MOTION (QUANTIS MOTUS)

'*The Quantity of Motion is the measure of the same, arising from the velocity and quantity of matter conjunctively.*'
'Quantity of Motion' is defined in DEF. II and 'Quantity of Mass' in DEF.I at page 2 of *the Principia*. It is different from motion:
Thus Quantity (amount or quantum or magnitude) of motion is the product of mass and velocity, also explained in the *Principia* just after the definition. Here Newton explained *The Quantity of Motion* in terms of velocity

Quantity of Motion = Quantity of Matter × velocity

$$= \text{mass} \times \text{velocity} = mv \qquad (5)$$

Newton did not write , Quantity of motion as velocity.

Quantity of Motion = Motion = velocity (speed) = v

Obviously Newton meant motion as velocity. Newton mentioned that motion was understood in terms of velocity earlier also. The Quantity of motion was not understood earlier hence Newton defined it specifically in Def. II. Newton gave reason for not defining motion that it was understood by previous scientists. Aristotle, Galileo, Descartes etc. has used term motion as velocity.

'Quantity of Motion' (product of mass and velocity of body) and motion are different; Newton discussed them separately at different places in the *Principia.* If the meaning of both terms were the same, then they would have been discussed together in the same line or sentence, and should have same units and dimensions. Further Newton should have defined motion in the definition section, but he only defined the Quantity of motion, and gave reason for not defining motion as it is already well understood. Newton did not define motion stating that it was understood earlier, but he defined Quantity of Motion as new or noble quantity, as it was not defined earlier. Newton himself distinguished between motion and quantity of motion as former is understood earlier and later is newly defined term. Thus motion and Quantity of Motion are different terms, hence cannot be regarded as replacement of each other in anyway. Both the quantities are not synonymous to each other.

Thus, the definitions of 'Quantity of Motion' and 'absolute motion', or motion, are entirely different. Newton regarded the motion as already understood, whereas Quantity of Motion defined in Def.II. Hence 'motion' cannot replace Quantity of motion; these are scientifically and conceptually different. Finally we can distinguish the Quantity of Motion and motion in the following way
The Quantity of Motion (quantity of matter × velocity)

Units : kg m/s Dimensions : $MLT^{-1}$

Motion ( Velocity)

Units : m/s Dimensions : $M^0LT^{-1}$

Raman has given explanation for Newton's Law as F=ma in the standard journal of physics education, in The Physics Teacher[17] that –
"*By 'motion' Newton meant 'quantity of motion' which he had defined as the product of mass and velocity, i.e. what he would call momentum. The crucial expression is 'change in momentum'. The usual tendency is to take this to mean 'rate of change of momentum*'"

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Firstly, the meanings of words or terms are well defined in equations and not stated by intuitive or tendentious way as stated by Raman[17] . Secondly, Raman has used terms like quantity of motion, motion, change in motion and rate of change of momentum; these terms have well defined and should not be misinterpreted. The terms like Quantity of motion, and motion are not synonymous. Raman should have mentioned about definition of 'Quantity of Motion' ( Def II) 'Quantity of Matter' (Def I) at pages 1-2 of *the Principia,* and reason given by Newton for not defining motion at page 9 , but these facts are not mentioned in the discussion by Raman. Thirdly Raman did not write mathematical equation based on the second law of motion (alteration is proportional to change in velocity i.e. F = $\Delta$v), however he stated the second law of motion in the article both in the Latin and English. Had he written mathematical equation for the law then the most of the issue would have been clear.

The most significant aspect is that F =ma was given by Euler in 1775, forty- eight after death of Newton. Apparently, Raman was unaware of scientific reality, and illogically established F = $\Delta$v and F =ma which is unscientific. Raman quoted this from the existing literature being taught since years in different part of the world. These books were published more than a century ago without peer review. Later on such teachings became scientific traditions. Raman did not give any scientific reason and wrote, '*The usual tendency is to take this to mean 'rate of change of momentum*'. The scientific laws and concepts are not established by assumptions, traditions and tendencies; the scientific logic is first and foremost requirement for the laws.

Needless to mention there is difference between change in velocity, $\Delta$v ( as in Principia's second law) and rate of change of momentum , $\frac{dp}{dt}$ ( as is Euler's law). Further the equality or replacement of change in velocity and rate of change of momentum is not permissible scientifically. Thus the *Principia*'s second law of motion, $F = (v-u)$, does not lead to $F = ma$, as alteration in motion ( $\Delta$v) can never justified as $\frac{dp}{dt}$. It is clearly unscientific that simple terms are misinterpreted. In such discussion the meaning of terms must be taken in scientific way not in tendentious way. It is not scientific at all. Thus, Raman's conclusion is inconsistent and not supported by any scientific logic. However Raman did not write any equation for the Principia's Second Law of motion . Had he written mathematical equation for the law then this issue would have been clear.

Had Newton given the mathematical equation in the *Principia* for the second law of motion then this issue would have not arisen. Consequently, scientists drew inconsistent similarities between the *Principia*'s second law of motion, $F= (v - u)$ and Euler's $F = ma$, then tried to justify that the equation for Newton's second law is $F = ma$, which is actually Euler's equation. F = (v-u) is never taught or interpreted in physics as if an unwanted child in physics.

Now there are some more scientific facts are helpful in understanding of the topic.

(i) Earlier in 14th century French philosopher Buridan (1300–1358) propagated Impetus Theory[18] and wrote equation similar to momentum.

$$\text{Impetus} = \text{weight} \times \text{velocity} \qquad (6)$$

However weight (mg) was defined precisely after 1686 when Newton published *the Principia* and acceleration due to gravity followed from the interpretation.

(ii) Also Leibniz (1646-1716) has coined the term vis viva[19] or

$$\text{living force} = mv^2 \qquad (7)$$

The units and dimensions of living force are that of energy.

(iii) The velocity was specifically defined in Jenning's book Miscellanea ( Definition II ) in *Latin* in 1721 as displacement divided by time[20]

$$V = S/t \qquad (8)$$

(vi) Whereas momentum was defined in Miscellanea ( Definition I ) as product of mass and velocity i.e.

P =mv (9)

Newton had ealier given it as 'quantity of motion' , and Buridan has given product of weight ( now , mg) and velocity as impetus as in eq.(6)

(v) The third edition of *the Principia* was published in 1726 but Newton did not mention or change any topic due to recent developments[21]
.

(vi) Euler defined force directly depending on mass, in the Proposition 17 of Mechanica Vol. II (1736 ) states[22]
:

"*The force of inertia of any body is proportional to the quantity of matter, upon which it depends*."

$F \; \alpha \; m$

$F = km$ (10)

If k is coefficient of proportionality. It is determined experimentally in existing literature.
(vii) Indirect or earliest hints of acceleration: In 1716, Jacob Hermann published a Latin text called "Phoronomia", meaning the science of motion[23] . He wrote the equation *dc* = *pdt*, where *c* stands for "celeritas" meaning speed or velocity, and *p* stands for "potentia" ( latin word ) , meaning force or power. Hence,

$$p = \frac{dc}{dt} = \text{power or force} \qquad (11)$$

But in today's notation it is acceleration (rate of change of speed or celeritas, dimensions, $M^0LT^{-2}$), which may be regarded as significant perception of acceleration. The dimensional analysis was initiated by Fourier[12-13] . in 1822, so this test was not applicable to equations i.e. up to 136 years after publication of *the Principia.* Thus, in view of this it is justified that an equation with dimensional formula $M^0LT^{-2}$ is written as force or power, at that time. However the dimensional formula for force is $MLT^{-2}$ and that of power $ML^2T^{-3}$. Whereas dimensional formula for acceleration is $M^0LT^{-2}$ . Thus Newton initiated physics but many other concepts were developed afterwards.
(viii) But Newton did not add or critically analyze these findings in the third edition of *the Principia* at all, and kept three laws of motion intact as in first edition i.e. for nearly 40 years.

## III. ALTERATION AND 'RATE OF CHANGE'

In the original definition of Newton's second law of motion, as given in all three editions of the *Principia* (1686, 1713, 1726), the word 'alteration' is used, but not 'rate of change of'. The third edition was published 40 years after the first. In fact in all three editions Newton did not change the definition of the three laws of motion at all, so he persisted with his own ideas. Newton may be been influenced lot by Galileo's demonstrations and law of inertia in this regard, the second part of first law of motion is Galileo's inertia. The first part of Newton's first law of motion is Aristotle assertion. However in these years some new developments took place, which were not incorporated by Newton.

In fact there is resemblance between Newton's second law and first law of motion. Newton's first law implies that body moves with uniform velocity in absence of force ( F =0 , v=constant ). The second law implies that the motion (velocity) changes when force acts i.e. alteration in motion is proportional to motive force, then it is due to force (F>0, then velocity is variable). Thus conceptually both laws give same meaning as second law implies that in absence of force i.e. F =0 (second law of motion), then body moves with uniform velocity. In simple words if force F=0, then second law of motion gives similar results like first law of motion. Any alternation in one law would have changed the status of the other. It is pertinent to mention that Newton's first law of motion follows from law of inertia established by Galileo.

Thus Newton may have decided not to change the definition of three laws of motion. Apparently he was influenced by Galileo's demonstrations and law of inertia, which he revamped as first law of motion. However the third law of motion is completely independent of first two laws. The definition, mathematical equations and

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interpretation of the laws must be such that they explain or accommodate the new developments. Thus the law must not be static or unilateral.

There are numerous examples in science that laws change with new experimental and theoretical findings. For example Aristotelian physics has been virtually abandoned. The dimensional analysis was initiated by Fourier in 1822 , thus discovery of units and dimensions has changed the older concepts drastically. Newton just initiated the beginning of physics by defining few terms. The beginning should not be regarded as end.

Newton estimated (assuming that propagation of sound in air is isothermal in nature ) the speed of sound in air as 280m/s, whereas experimental value is 330m/s. Thus Laplace corrected Newton's equation (establishing that propagation is adiabatic in nature), so that exact value is obtained. Newton's corpuscular theory of light has been replaced by Huygen's wave theory of light . Now quantum theory of light has been formulated. Newton's law of cooling states that heat loss of a body is directly proportional to the difference in the temperatures between the body and its surroundings provided the temperature difference is small. A correction to Newton's law concerning larger temperature differentials was made by Dulong and Petit. Further Newton's third law (establishes universal equality between action and reaction) has been critically discussed to assess that the effects of inherent characteristics, nature and composition of bodies, hence has been theoretically generalized[5]. The law should be such that it may accommodate or explain all theoretical and experimental facts, as they are known or discovered in future. Einstein put forth static theory of universe, and insisted with this till mid 1930s, the same has been replaced by expanding theory of universe. Needless to mention that for nearly 18 centuries scientists believed that Earth is stationary and Sun revolves around it. There are many such examples that when established laws are changed after their long applications.

The meaning of alternation in Oxford Dictionary is *The action or process of altering or being altered (changed).* 'Alteration' means change in two stages or states ($v_2$-$v_1$). Thus eq.(1) is mathematical equation based on the definition
.

$$F = \Delta v \qquad (3)$$

However, in the current literature, the 'rate of change of' is used, instead of 'alteration'; which is not justified. Both alteration (difference) and 'rate of change' (variation with respect to time) i.e. derivative are entirely different, not synonymous.

Change in velocity = $\Delta$ v

Units m/s, Dimensional equation, $\Delta v = M^0LT^{-1}$

Rate of change of momentum = $\frac{d(mv)}{dt}$

Units kg m/s$^2$, Dimensional equation , d(mv)/dt = $MLT^{-2}$

Thus $\Delta$ v or $\frac{d(mv)}{dt}$ are conceptually and mathematically entirely different. These terms cannot be regarded in science as replacements for one another. These terms have different units and dimensions. Newton did not write 'rate of change of ' in his work. So it is not logical to replace 'rate of change of momentum' with 'alteration in motion' or velocity, as these are also not synonymous. Further quantity of motion and motion are different. The quantity of motion is product of quantity of matter and velocity.

Newton has used phrase in second law of motion as *The alteration of motion,* which means change in velocity i.e. ($v_2$-$v_1$). Had Newton used phrase *The alteration of Quantity of Motion*, then it would have been ($mv_2$-$mv_1$) or m$\Delta$v. Had Newton used phrase the rate of change of *the alteration of Quantity of Motion*, then it would have meant mdv/dt or ma . Further the alteration in velocity can never be equal to rate of change of momentum.

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In the Principia Newton categorized quantities in two ways i.e. already known quantities like time, space, place, motion etc. , which were earlier known to scientists and well understood. So Newton did not define these quantities as stated by him at page 9 of *the Principia*.

Secondly, Newton defined new quantities in the beginning of *the Principia* at pages 1-8, in definition section such as (quantity of matter, quantity of motion, innate force of matter, impressed force, centripetal force, etc.). But it is not prudent to conclude that Newton knew everything, he discovered everything there was nothing to be further known .

Newton neither wrote acceleration (change in velocity/ change in time, $M^0LT^{-2}$, $m/s^2$) in new definitions (quantity of matter, quantity of motion, innate force of matter, impressed force, centripetal force, etc.) nor mentioned it in existing definitions such as time, space, place and motion. Thus acceleration was not known in the time of Newton. He maintained that existing definitions are well known and do not need any explanation. If quantity acceleration was known to Newton, he would surely have explained it here, either in new or existing definitions. Thus acceleration was not discovered in Newton's time, hence not discussed anywhere in the *Principia.* It was unknown at Newton's time. So it was impossible for Newton to write equation, $F = ma$. The equation $F = ma$ is not mentioned in the *Principia.* This equation was written for first time, F=ma by Euler in 1775.

Further it is univocally clear that the acceleration and second derivative were not discovered in Newton's time[24]
.

Thus in Newton's time the laws were formulated qualitatively. The formulation of equations and mathematical interpretations were next phase of development of concepts. Finally mathematical predictions were experimentally confirmed. The units and dimensions were defined afterwards, and dimensions of force are based on F=ma. It is concluded independently that Newton did not give F=ma in *the Principia.*

According to Stanford Encyclopedia of Philosophy[21] — "The modern *F=ma* form of Newton's second law nowhere occurs in any edition of the *Principia* even though he had seen his second law formulated in this way in print during the interval between the second and third editions in Jacob Hermann's *Phoronomia* of 1716. Instead, it has the following formulation in all three editions: *A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed*." Further American journal of Physics[25] states, "But there is nothing in the Principia's second law about acceleration and nothing about rate of change." Thus it was impossible for Newton to write F =ma.

"The obvious question with the second law is what Newton means by 'a change in motion'. If he had meant a change in what we call momentum — that is, if he had meant, in modern notation[21], $m\Delta v$ — the proper phrasing would have been "*a change in the quantity of motion"*
.

"If this way of interpreting the second law seems perverse, keep in mind that the geometric mathematics Newton used in the *Principia* — and others were using before him — had no way of representing acceleration as a quantity in its own right. Newton, of course, could have conceptualized acceleration as the second derivative of distance with respect to time within the framework of the symbolic calculus. This indeed is the form in which Jacob Hermann presented the second law in his *Phoronomia* of 1716 (and Euler in the 1740s). But the geometric mathematics used in the *Principia* offered no way of representing second derivatives[21] "
Further it must be noted that Newton's first and second laws of motion are meant for linear motion, it would be arbitrary if the same ( F=ma) is derived from circular , periodic , oscillatory or random motion. Such attempts are never justified, and Euler's derivation of F =ma should not be ignored. In geometrical interpretation the circumference, distance of points from centre, chord , radius, arc etc. can vary, so results can vary.

Newton never gave any equation for his laws in *the Principia,* had Newton given mathematical equations for the laws then there would have been no need of this discussion[11] .Thus there is no possibility that Newton ever gave equation F =ma. Thus Newton never defined F =ma (now taught as Newton's Second Law of Motion) in any edition of *the Principia,* which was actually given by Euler.

Further, it was natural for Newton to stay with the established tradition[21] of using a length as the measure of the change of motion produced by a force, even independently of the advantage this measure had of allowing the law to cover both discrete and continuously acting forces (with the given time taken in the limit in the continuous case). But the scientific conclusions must not be drawn on the basis of old traditions. The length and velocity have different dimensions ($M^0LT^0$ and $M^0LT^{-1}$ and units i.e. m and m/s), hence both can never be compared. The length and change in velocity are entirely different concepts, thus length cannot be used to measure the change in velocity. If conclusions are drawn on the basis of such similarities then these are inconsistent. Thus such conclusions are drawn by on the basis of assumptions and suppositions are not scientifically logical[11,17,25]

The irony is that Newton started initiation of physics, at that time physics appeared to be branch of mathematics, geometry and natural philosophy. So the laws were not expressed in terms of equations, as equations were not necessity or perceivable at that time in further explanation of the law. Now when laws are expressed in terms of equations, newer and newer facts are emerging in understanding the concepts. The equation based on second law of motion i.e. F = (v-u), is not taught or discussed anywhere, whereas as equation given by Euler F =ma is taught as Newton's second law of motion all over the world. The reason is very simple, the equation F =(v-u) was never written and interpreted by Newton, it was written afterwards. But even now it is not taught anywhere, but F=ma is regarded as second law of motion.

In 1775 Euler gave equation of force F =ma, which was readily used by scientists as Newton's second law of motion, one of the reasons may be that at least both equations are of force. However in RHS the meanings of the variables in both equations is entirely different. Further Newton was credited with Euler's discovery after the death of both scientists. Otherwise Euler would have raised authorship issue like contemporary Robert Hooke. Or even Newton had pointed out that it is the meaning of the law he had defined. Further meanings of various terms in *the Principia* may have not been clearly understood. Due to lack of communications and technological facilities in earlier days the scientific laws of various scientists remained confined to certain regions only. It is well known that scientific correspondence between and Liebniz (Germany) was facilitated by boats or ships. It took months when scientists got reply from other.

Strictly speaking the process of peer review before publication started in 20th century. The reason may be that in earlier days, the research articles were in short supply and journals or publications may be localized in limited regions in local languages only with limited readership comparatively. When modes of communications and transport enhanced, scientists came together to set up certain rules and regulations of research publications, the peer review was one of them.

Even most of Einstein's work is published without peer review. For example, Max Planck (1858-1947) was editor of *Annalen der Physik* w.e.f. 1907-1943. It is clearly stated that in these times, peer-review was not yet standard. Einstein, for example, just sent his manuscripts to Planck who then subsequently published them [26]. In 1948 the 9th Confe´rence Ge´ne´rale des Poids et Mesures held , and adopted the name "Newton" for the unit of force in resolution7 [27] The progress of scientific doctrine is gradual not sudden or abrupt. Further the theme of discussion is that the methodology or procedure of acceptance of scientific laws is variable or relative. The scientific community is open minded to discuss the same, which is spirit of science. Newton was credited with Euler's discovery (F =ma) after deaths of both Euler and Newton.

**Newton did not write any equation for three laws of motion and law of gravitation.**

Newton did not write any equation for second and third laws of motion including for law of gravitation. Thus, revisiting the laws logically and critically has always been useful. Whereas Newton stated the law of gravitation in form of Propositions 1-XIII (Book III), and the equation ($F = GmM/r^2$) was not written by Newton. Thus method of pedagogy, epistemology and research was entirely different in Newton's days. The Law of Gravitation is based on the following Propositions :

**Proposition I :** That the forces by which the circumjovial planets are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to Jupiter's centre; and are reciprocally as the squares of the distances of the places of those planets from that centre.

$$(\text{force } \alpha \;\; r^{-2})$$

**Proposition II :** That the forces by which the primary planets are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to the sun; and are reciprocally as the squares of the distances of the places of those planets from the sun's centre.

( force $\alpha$ $r^{-2}$)

**Proposition III :** That the force by which the moon is retained in its orbit tends to the earth; and is reciprocally as the square of the distance of its place from the earth's centre.

( force $\alpha$ $r^{-2}$ )

Proposition IV That the moon gravitates towards the earth, and by the force of gravity is continually drawn off from a rectilinear motion, and retained in its orbit.

( force $\alpha$ $r^{-2}$ )

**Proposition V:** That the circumjovial planets gravitate towards Jupiter; the circumsaturnal towards Saturn; the circumsolar towards the sun; and by the forces of their gravity are drawn off from rectilinear motions, and retained in curvilinear orbits.

**Proposition VI :** That all bodies gravitate towards every planet; and that the weights of bodies towards any the same planet, at equal distances from the centre of the planet, are proportional to the quantities of matter which they severally contain.

( weight $\alpha$ quantity of matter)

**Proposition VII:** That there is a power of gravity tending to all bodies, proportional to the several quantities of matter which they contain.

( Power of gravity $\alpha$ quantity of matter)

**Proposition VIII :** In two spheres mutually gravitating each towards the other, if the matter in places on all sides round about and equi-distant from the centres is similar, the weight of either sphere towards the other will be reciprocally as the square of the distance between their centres.

( weight of sphere $\alpha$ $r^{-2}$ )

**Proposition IX :** That the force of gravity, considered downward from the surface of the planets, decreases nearly in the proportion of the distances from their centres.

( force of gravity $\alpha$ $r^{-2}$ )

**Proposition X:** That the motions of the planets in the heavens may subsist an exceedingly long time.

**Hypothesis I :** That the centre of the system of the world is immovable.

**Proposition XI:** That the common centre of gravity of the earth, the sun, and all the planets, is immovable.Proposition XIII The planets move in ellipses which have their common focus in the centre of the sun; and, by radii drawn to that centre, they describe areas proportional to the times of description. ( Kepler's first law )

**Kepler's laws of motion:** Newton defined in various Propositions for various planets e.g. circumjovial planets, circumsaturanal planets, circumsolar planets, they tend to Jupiter, Saturn and Sun respectively. Also Newton mentioned about inverse square law in various propositions.

Further Newton *re-quoted* Kepler's first law of motion (planet revolve around the sun) and Kepler's second law of motion in Proposition XIII, which existed since 1609. But Kepler has neither stated cause of motion for planets nor inverse square law[28] . Kepler's laws are based upon Tycho Brahe's observations, who studied effects (positions of planets at various times) without finding out the cause. Kepler gave scientific laws based of the existing observations of Tycho Brahe, and Newton further mentioned about the cause of motion of the planets. So science is like lighting one lamp to other.

It has been pointed out that the understanding of any law is only complete when mathematical equations based on it are experimentally confirmed. In the above Propositions 1-XIII out of total Propositions I-XLII, in the Book III and in XCVIII Proposition I of Book I of *the Principia* ; the force of attraction is proportional product of masses of bodies (mM) is never written. The Book I (*On the motion of bodies )* and Book II (motion through resisting mediums) of *the Principia* deal with different concepts and topics. The various Propositions I-XII implies that force is proportional to inversely square of distances between bodies. In Proposition V1 Newton stated that weight of planet is proportional to quantity of matter (mass, as defined in Definition 1 of *the Principia*). Newton did not give any equation in *the Principia* for law of gravitation. However in the modern physics equation for law of gravitation is given by

$$F = GmM/r^2 \tag{12}$$

where G is universal gravitational constant. The value of G was determined by Cavendish in 1798 i.e. 112 years publication of *the Principia*.

Here it is pertinent to quote - Hooke's correspondence with Newton during 1679–1680 not only mentioned this inverse square 'supposition' for the decline of attraction with increasing distance. More significantly in Hooke's opening letter to Newton, of 24 November 1679, described an approach of "compounding the celestial motions of the planets of a direct motion by the tangent and an attractive motion towards the central body[29] ".

Sir Isaac Newton acknowledged Christopher Wren, Robert Hooke and Edmund Halley in this connection in the Scholium to Proposition 4 in Book 1. Further Newton maintained that Hooke may have given idea but he fully developed it physically by giving evidences and mathematical calculations for solar system.

Now it would be important to investigate that whether between 1666 and 1679, did Newton write or publish 'inverse square law" in any form or not. If he did then credit will go to Newton for discovering the "inverse square law". If Newton did not, then Robert Hook would get the credit.

There are 98 Propositions in Book I and 42 Propositions in Book III. In any of the Propositions in Book I or Book III , Newton did not write force of attraction between bodies as product of masses of bodies (mM), however Newton frequently motioned that force of attraction varies as inversely proportional to square of distance. But in mathematical equation of law of gravitation, the force of attraction is regarded as product of masses of bodies and inversely proportional to square of distance. It must be critically analyzed from the literature, whether Newton, Hook or some other scientist stated that force is proportional to product of masses of bodies. The value of G in mathematical form of law of gravitation was calculated by Cavendish in 1798. The thorough analysis of documents in libraries of universities of Cambridge and Oxford will be helpful in this regard.

## IV. EULER GAVE FOUR EQUATIONS OF FORCE RELATED WITH ACCELERATION AND MASS

Basically Euler has given four equations of force F=ma/n (1736), F =2ma (1750) , F=ma/2g (1765) and F=ma (1775) at different stages. In Euler's time (1703-1783) scientific concepts and terminology were entirely different i.e. at very immature stage. Further if vast literature of Euler (nearly 900 articles, scientific documents and books) is critically studied then more equations may be directly or indirectly possible[9] . Only few articles have been translated to English thus analyzed by wider audience. The more useful results are expected if Euler's scientific work is critically analyzed [3] and translated to other languages. Euler's various equations of force are completely independent of *the Principia's* second law of force i.e. *–the alteration in motion is proportional to force* i.e. F = (v-u).

(a) In 1716, Jacob Hermann published a Latin text called "Phoronomia", meaning the science of motion[23] and wrote equation dc=pdt. Newton did not apply or mention this equation in *the Principia* or any of his following work. However Euler utilized this equation and applied the equation (may be even empirically) for various systems to calculate the force. This is difference between Euler's and Newton's approach. In 1736, Euler wrote

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equation of potentia (p) meaning force or power which has resemblance with eq.(4) i.e. dc=pdt,

$$dc = \frac{npdt}{m} \quad \text{or} \quad F = \frac{1}{n}\text{ma} \tag{13}$$

where m is mass, c is velocity, F is force , t is time and where n is constant and depends upon unity of measure[30-31] . Euler has already associated force with mass[22] , similarly force depends upon mass in eq.(13). By unity of measure we mean, unit of measurement. Euler[18,29] , used two primary or fundamental units L (length) and F(force), thus coefficient/constant of proportionality is 2. Now co-efficient are determined experimentally. Thus Euler began the acceleration dependent of force and finally he reached at equation F=ma in 1775. The systems of primary units L (length)-F(force) –T(Time ) and L (length)-M(mass) –T(Time ) were introduced in the following century. From eq.(6) Euler was able to derive all differential equations necessary to describe the motion of a point-mass.

(b) In *Mechanica*, however, Euler used an intrinsic coordinates system. He decomposed speeds and forces according to directions that depended upon the intrinsic nature of the problem. In these papers, Euler used an extrinsic references frames (a system of three orthogonal Cartesian axes) and formulated the following equations of force[32-33]
:

$$2Mddx=Pdt^2,\ 2Mddy=Qdt^2,\ 2Mddz=Rdt^2, \tag{14}$$

$$\text{or} \quad P = 2M\frac{d^2x}{dt^2},\ Q = 2M\frac{d^2y}{dt^2},\ R = 2M\frac{d^2z}{dt^2} \tag{15}$$

where *M* is the mass and *P*, *Q*, and *R* the components of the force on the axis (the coefficient 2 depended on the unity of measure , and Euler has chosen two primary units ).

$$F = 2M\frac{d^2s}{dt^2} = 2M\frac{dv}{dt} = 2\frac{dp}{dt} = 2Ma \tag{16}$$

In view of it (dependence upon unity of measure ) coefficient, n is 2 in eq.(6) i.e. F=ma/2. For completeness all equations have to be mentioned.

(c) Later, in 1765, Euler introduced the concept of moment of inertia of a rigid body and decomposed the motion into the rectilinear motion of the centre of mass and proposed equation[34] .

$$F=\frac{Ma}{2g} \tag{17}$$

The value of coefficient of proportionality establishes numerical equality between LHS and RHS of equation. Now a days its value is determined experimentally taking in account all possible values of parameters; however situation was different in the earliest days when physics was in inception stage , and even no units, dimensions and no systems of units were defined. Initially laws of physics were defined just philosophically without any mathematical basis i.e. qualitatively. The earliest footprints change to final roadmap. There are many coefficients of proportionalities in the existing science which are determined experimentally. For example, coefficient of viscosity, coefficient of thermal conductivity, coefficient of thermal expansion, coefficient of friction, Hubble's constant ( actually coefficient ), electrochemical equivalent , Young's modulus ( coefficient in elasticity ) , coefficient in Coulomb's law , Stefan's coefficient , various coefficients in C.F. von Weizsacker's formula etc. etc. Also coefficients exist in various equations which illustrate the various laws in science, thus coefficients are not only confined to physics.

(d) Further in 1775, he completed the construction of general equations of dynamics by formulating a system of six equations determining the motion of any body, which (except for an additional coefficient) he wrote in this way [2].

$$P=\int dM \frac{d^2x}{dt^2}, \quad Q=\int dM \frac{d^2y}{dt^2}, \quad R=\int dM \frac{d^2z}{dt^2}$$

Or in general, $F = \int dM \frac{d^2s}{dt^2} = ma$ (2)

As F=ma is the last or the simplest available equation given by Euler is used in calculations. This equation is known as Newton's second law of motion.

## V. F =MA CANNOT BE DERIVED FROM CURVED MOTION, ARBITRARILY.

Initially, the geometrical illustrations were used to express various concepts. The analytic and algebraic equations were developed afterwards and Euler developed the mechanics with analytical equations. It is crystal clear that Newton did not give F =ma in *the Principia*, also it can never be recovered from curved (circular or elliptical or any irregular) motion described in *the Principia*. The various Propositions which critics[35] illogically and unscientifically maintain that these may lead to second law of motion are discussed here.

Newton, like his predecessors, expressed some of such doctrines mostly in abstract form and geometrically. But now science is synonymous with exactness. These were expressed further and illustrated by Propositions (a statement that affirms or denies something and is either true or false, it is not experimentally established principle using mathematical equations), Theorems (A proposition deducible from basic postulates), Lemmas (a helping theorem), Scholiums (an explanatory note), explanations etc. Currently a law is stated, and then mathematical equations are written. Afterwards if the mathematical predictions of equation are experimentally justified, then law is regarded as experimentally justified.

For laws of motion, Newton was influenced by Galileo's inertia and related observations, and first two laws of motion directly draw support from Galileo's observations. Newton's Third law had resemblance with Descartes third law of motion. Whereas for planetary motion Newton quoted Kepler's laws in slightly different way along with inverse square law of motion. Further Kepler's law is based on observations of Tycho Brahe (1546-1601). Thus silent contributor for planetary laws of motion is lifelong observations of Tycho Brahe.

Even Proposition 1 of the Book I and Proposition XIII of Book III (discussed below) are nothing but Kepler's second of planetary motion i.e. area traversed by planet around the Sun is dependent on time. Newton has simply taken them from existing literature. Kepler's first law states that planets revolve in elliptical orbits. In the various Propositions I-X in Book 1, Newton had written that planets keep their orbits (Kepler's first law states that planet move around the Sun in elliptical orbits). Thus the perception that these planets revolve around the sun was existing before Newton. It was simply re-quoted Newton from the existing text. The force varies as reciprocal of square of distance, is mentioned frequently in propositions but Hooks too have contested the priority of the law.

Kepler's first two laws were published in 1609 (i.e.77 years before publication of Newton's *Principia*. ) in the Kepler's book *Astronomia nova* (*New Astronomy).* Thus basis of planetary motion was established by Kepler on the basis of observations of Tycho Brahe (1546-1601). Newton gave Propositions 1-XIII in Book III of *the Principia*, describing motion of planets and equation for law of gravitation $F = G\ mM/r^2$ was written later on. F =ma explains linear motion whereas these laws are interpreted in terms of circular or elliptical or curvilinear motion in orbits. Both types of motion are entirely different. Newton's second law of motion, F=ma is not originated from this discussion at all.

Thus Newton's quotation is justified here that his work is based on two giants is completely justified here. His laws of motion are influenced by Galileo and law of gravitation on the laws of Kepler which are based upon observations of Tycho Brahe.

The real observation is that Newton did not mention in the law of gravitation that force of attraction is proportional to product of masses of planets (Mm), and Hooke contested the priority of inverse square law ($1/r^2$). On the other hand fact that planets revolve around the Sun are given by Kepler (based on the observations of Tycho Brahe). Thus Newton elaborated Kepler's law; as law of gravitation that why planets

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revolve? Newton did not write equation F = GmM/$r^2$, which was written afterwards by other scientists. Cavendish determined the value of G in 1798 i.e. 112 years after publication of first edition of *the Principia.*

Newton used the geometrical illustrations to express concepts without mathematical equations. But now days mathematical equations are experimentally confirmed, only then law is regarded as finally established. The analytic and algebraic equations were developed afterwards and Euler developed the mechanics with analytical equations. It is crystal clear that Newton did not give F =ma in *the Principia*, also it can never be recovered from curved motion described in *the Principia*. The various Propositions which critics illogically mention that these may lead to second law of motion are discussed here.

The basic reason for these illogical deductions is non-discussion of the fact that Leonhard Euler derived F =ma and published in 1776, in the journal Commentarii academiae scientiarum Petropolitanae published by St. Petersburg Academy. Now the paper in which F =ma is derived is available in the website of Mathematical Association of America, Washington, article Index No E479 at page 223.

**Prop.I, Prop. VI and Prop. LXVI of Book I of the Principia**

Prop. I (expresses nearly Kepler's law of planetary motion), Prop.VI (describes centripetal force as versine or $2\sin^2(½\theta)$, i.e. $1 - \cos\theta$, and inversely proportional to square of time) and Prop. LXVII (describes decrease in force of attraction in three bodies of different sizes placed at different distances) individually or in combined form do not lead to *F = ma*. These Propositions deal with circular or elliptical motion not with linear motion. Newton had given these Propositions for entirely different purposes. These propositions do not involve mass, m of body in equations, without mass, F=ma cannot be perceived. They do not lead to *F = ma* individually or jointly.

The basic reasons for these illogical deductions are non-discussion and non-inclusion of the facts that Leonhard Euler[8] derived F =ma and published by Saint. Petersburg Academy in 1776. Now the paper in which F =ma is derived is available in the website of Mathematical Association of America, Washington, article Index No E479 at page 223. All the facts are analyzed in view of Euler's discovery of F=ma which has been neglected in the presvious discussions.

**Proposition 1 (Book I)**

The areas, which revolving bodies describe by radii drawn to an immovable centre of force do lie in the same immovable planes, and are proportional to the times in which they are described.

Kepler's 2nd law (law of equal areas) can be quoted as:

"a line connection the Sun and a planet (called the radius vector) sweeps out equal areas in equal times."
Further Newton had repeated Proposition 1 (Book I) as Proposition XIII (Book III)
Prop. I (resembles Kepler's second law based upon observations of Tycho Brahe) implies the areas described by planets around the centre are proportional to the time taken to describe them:
*Area described by revolving body = K. time in which area is described.*

*K* is coefficient of proportionality, which is experimentally determined.

**Proposition VI (Book I)**

*In a space void of resistance, if a body revolves in any orbit about an immovable centre, and in the least time describes any arc just then nascent; and the versed sine of that arc is supposed to be drawn bisecting the chord, and produced passing through the centre of force: the centripetal force in the middle of the arc will be as the versed sine directly and the square of the time inversely.*

This Proposition assumes that centre of the orbit is immovable. But the sun is not immovable , it moves around the centre of Milky Way with a speed of about 240km/s. The distance from the center of our Galaxy to the Sun is about 26,000 light years. The versine or versed sine, versin($\theta$), is a trigonometric function equal to $1 - \cos(\theta)$, or $2\sin^2(½\theta)$. Proposition VI implies,

$$F \text{ (centripetal force in the middle of arc)} = K\frac{2\sin^2(½\theta)}{t^2} \qquad (18)$$

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Thus mathematically in eq.(18) the centripetal force is independent of mass and radius. It is clear that F =ma does not follow from this discussion, it was derived by Euler in 1775. Acceleration is rate of change of velocity. In circular motion velocity is constant. The arbitrary arguments are not allowed in science.

**Proposition LXVI. (Book I)**

*If three bodies whose forces decrease in a duplicate ratio of the distances attract each other mutually; and the accelerative attractions of any two towards the third be between themselves reciprocally as the squares of the distances; and the two least revolve about the greatest; I say, that the interior of the two revolving bodies will, by radii drawn to the innermost and greatest, describe round that body areas more proportional to the times, and a figure more approaching to that of an ellipsis having its focus in the point of concourse of the radii, if that great body be agitated by those attractions, than it would do if that great body were not attracted at all by the lesser, but remained at rest; or than, it would if that great body were very much more or very much less attracted, or very much more or very much less agitated, by the attractions.*

In first part thereof this Proposition states that forces decrease in duplicate ratio of the distances and attract each other mutually,

$$\left(\frac{F_1}{F_2} \propto \frac{r_1^{\,2}}{r_2^{\,2}}\right) \quad \text{or} \quad \frac{F_1}{F_2} = K\frac{r_1^{\,2}}{r_2^{\,2}} \tag{19}$$

If we keep three magnets (bodies) of different strengths, different masses, placed on the ground, the smaller two do not revolve around the larger. They may even repel each other. This Proposition does not involve acceleration $a = (v - u)/(T - t)$, and hence impressed force, F = ma.

Thus from various propositions we get,

*Area described by revolving body = K. time in which area is described*

.

$$\text{F(centripetal)} \propto 2\sin^2(½\theta) \quad \text{(versed sine)}, \qquad F\text{(centripetal)} \propto \frac{1}{t^2}$$

$$\text{or} \quad \text{F(centripetal)} = K\frac{2\sin^2(½\theta)}{t^2} \tag{20}$$

Thus, mass does not come into the picture. It does not lead to F =ma, each one describe different problems.

These equations do not lead to F =ma. In science unrealistic and illogical deductions are not allowed. It is crystal clear that reason for such discussion is that scientists did not take in account that the equation F=ma , was Euler has already published paper. Newton's Second law of motion as given in *the Principia*, is F = (v-u) is not taught or discussed anywhere as in today's context it not useful equation. For one reason or other scientists wanted to keep alive Principia's second law of motion hence they misinterpreted it, even distorting its definition. In similar way earlier scientists took considerable time to accept Huygens's wave theory of light which pointed out limitations of Newton's corpuscular theory.

## VI. HOW CREDIT OF F =MA MAY HAVE BEEN GIVEN TO NEWTON?

Next question is how and when Euler's equation F=ma was used as Newton's second law of motion? In view of it we considered two books published in 1871 and 1934; however it is added there are many such books. The final conclusions must be drawn after consulting all relevant books. In a book[36] written by J H Evans, *The First Three Sections of Newton's Principia* , it has been remarked as Cambridge College and School textbook published in 1871, carries the second law of motion as stated exactly in *the Principia*. Thus in a standard textbook and quote original law without distortion, even after 96 years of enunciation of F =ma. Thus misinterpretation of definition of *the Principia*, took place after 1871. Whereas in another book[37] titled *Newton's Principia* published in 1934 written by Cajori , the original form of second law is quoted same as in *the Principia*. But in its appendix attempt has been made to misinterpret the law taking 'motion' and 'quantity of motion' as same. It is already justified these two are not synonymous. Also 'alteration' and 'rate of change of

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'motion' are regarded as same whereas two are not entirely different terms. Consequently, it is assumed that F= $\Delta v = \frac{dp}{dt}$ =m $\frac{d^2x}{dt^2}$ , which is unscientific.

The same arguments are re-quoted by Raman[17] in the *Physics Teacher* and in all the textbooks all over the world. The standard text books gives same reasons to change F=(v-u) to F =ma . The comparison of existing literature especially present at libraries of universities of Cambridge and Oxford, can be useful in understanding the fact how and when the credit of F =ma was given to Newton.

Euler was born in Switzerland Basel, he got his education there. In 1727, he migrated to St. Petersburg, Russia. Euler's eyesight worsened throughout his mathematical career. In 1738, three years after nearly expiring from fever, he became almost blind in his right eye. Concerned about the continuing turmoil in Russia, Euler left St. Petersburg on 1741 to take up a post at the *Berlin Academy*. Then in 1766 Euler accepted an invitation to return to the St. Petersburg Academy. Euler later developed a cataract in his left eye, he became completely blind in 1766 i.e. at age of 59. A fire in St. Petersburg in 1771 cost him his home, and almost his life. Then he wrote papers in which F =ma was written in Russia in 1775 for three dimensional motion[18] . His most of works are still in *Latin*, which are being voluntarily translated by scientists and mathematicians.

On the other hand, at age of 27 in 1669 Newton became Lucasian Professor of Mathematics. He was elected in 1672 Fellow of Royal Society of London. In 1696 Newton was offered position of Warden of Mint which he accepted, in 1700 he was appointed master of Mint and in 1701 Newton was elected to Parliament by Cambridge Senate. Finally in 1703 elected President of Royal Society London and he occupied the post till his death. In 1705 Newton was knighted by Queen Anne in Cambridge. Thus Newton experienced highest administrative and academic positions for nearly last 25 years of his life. Newton was resident of powerful nation England, whereas Euler Swiss citizen spent his last 56 years of life in Russia and Germany mainly. Euler remained completely blind in last 17 years of his life. This is the contrast between two who was given undue credit of F=ma, and one who is deprived of the same.

Historically the reason for this lapse may be that Euler's work was not well complied as he worked in Switzerland, Germany and Russia. Whereas Newton was based at University of Cambridge (the most reputed since beginning in 1226) and his work is well complied in *the Principia*. Initially Newton's work was taught in England, then adopted worldwide in due course of time, as British empire expanded. Now Euler's work is well complied by Mathematical Association of America and available online[9] . Nearly 150 articles of Euler are translated to English out of 866. Thus newer facts are coming in the picture. Then scientists found Euler's equation F=ma exceptionally useful but may not be aware of actual originator. Thus made arbitrary changes in definition of Principia's second law of motion in textbooks so that its mathematical form appears like F =ma. Euler has independently given four equations of force.

It must be noted that Newton's Principia has been studied by many scholars and large number of books are available. But here our aim is to critically understand the Newton's original version of the *Principia* , thus English translation of *the Principia* by Andrew Mott and as far as possible the original Latin version of *the Principia* are consulted. Although Andrew Mott had translated *the Principia* yet in some cases it also appears that text has been elaborated as in modern versions of *the Principia*. However it requires separate study. In the current discussion our aim is to discuss Newton's original work not others' commentaries of *the Principia.*

During Victorian age Great Britain was the largest imperial power in the world and produced around 30 percent of the world's total industrial output. Out of 193 countries currently recognized by the UN, Great Britain[38] invaded 158 . That's an amazing 82% !. Whereas Switzerland has only invaded in 1815 another state France.

Newton belonged to such a great country, his works were published and circulated very quickly in the world through British empire, whereas Euler belonged to Switzerland ,he worked in Germany and Russia. Euler's spent last 17 years of his life as completely blind. Euler's paper[9] E479 in which F =ma was given was originally published the journal Novi Commentarii[8] ,which was published by St. Petersburg Academy in 21 volumes between 1747 and 1778 . The scientists who credited F =ma to Newton misinterpreting Second Law of Motion may not be aware of it.

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The most unfortunate fact was that Euler remained completely blind for last 17 years of his life, so his work was not properly complied and circulated through widespread journals. Another major factor is that Euler published more than 866 articles, so these articles have to be thoroughly studied for proper analysis and priority issues. Whereas Newton's book the Principia was well known and well circulated. There is similarity between works of legends that both deal with force ( i.e. Newton's second law and Euler's F=ma), in LHS but RHSs are entirely different. Further equation F =ma has exceptional mathematical adaptability; scientists might have thought it is given by exceptionally genius scientist Sir Isaac Newton of a powerful nation, England. Further Cambridge and Oxford are the most celebrated places of knowledge since their inception to till date. At that time Euler's articles may be obscure. When an idea is accepted globally, even may not be fully correct, then it is uphill task to amend the same when scientist is like Newton and nation is like England.

To error is human; so even by mistake some scientists stated that Newton's second law implies equation F =ma, then it was regarded as true. Further more serious mistake was that scientists tried to justify that mathematical equation for Principia's definition of second law is F=ma. To erreor is human , to correct is divine. Newton had not given any equation for his law, it must be noted a significant factor. It provides ample chances for misinterpretation or misunderstanding. Moreover it took place after deaths of Newton and Euler, none could object. The scientists who credited F =ma to Euler, may not be aware that Euler has published the equation F=ma in 1776 in the journal Commentarii academiae scientiarum Petropolitanae published by St. Petersburg Academy.

However Robert Hooks contemporary Newton living in England , raised the issue that he is originator of idea that force of attraction decreases as inverse square of distance. Further, German Leibniz disputed some aspects of discovery of calculus. Under the colonial rule of Great Britain, it was impossible for someone to challenge the authority in any way. As discussed earlier there was nothing like peer review in those days. The process of peer review started nearly in the mid 20th century, even Einstein's most of work was published without peer review. But currently situation is different we can analyze the facts and put forth all possibilities together. Thus credit of the discovery of equation F=ma (taught as Second Law of Motion) should be given to Euler, and that of F = (v-u) to Newton.

## VII. NEWTON'S TWO QUOTATIONS

The greatest but the humblest scientist expresses his philosophy or approach towards science in two quotations :
*'I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the sea-shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.'*

Also,
*If I have seen further than others, it is by standing upon the shoulders of giants.'*
Thus it is true that science is lighting one lamp from the other. These are justified from the following discussion.

**Influences of Kepler, Aristotle, Galileo and Descartes**

It is evident that Newton's law of gravitation and three laws of motion are influenced by work of his predecessors.

**Kepler's laws of motion:** Kepler (1571-1630) stated two laws of the planetary motion i.e. all planets revolve around the sun in elliptical orbits ( differences between semi major axis and semi minor axis is small) and all planets sweep out equal areas in equal times (law of areas). The shape of earth's orbit is spheroid rather than ellipsoid. It is the same thing Newton has assumed that all planets revolve around in their proper orbits around the sun. In Book III of the Principia, in various propositions Newton has mentioned the planets revolve around the sun in their proper orbits which are regarded as nearly circular (centripetal force i.e. $F = mv^2 /r$ is applied).

Needless to mention that Kepler's laws are further based on observations of his mentor Danish astronomer Tycho Brahe (1546-1601 ). In the various propositions of Book III of the Principia Newton do not mention that force of attraction between two bodies is proportional to product of masses of two bodies. As far as inverse square dependence of law of gravitation is concerned, it is contented by English engineer and scientist Robert Hooke. Sir Isaac Newton acknowledged Christopher Wren, Robert Hooke and Edmund Halley in the Scholium to Proposition 4 in Book 1 regarding the discovery of 'inverse square law'. Thus contribution/claim of others

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needs to be investigated whether it was just to dilute Hooke's contribution or other scientists too really contributed to it. In fact Newton's first two laws are based on perceptions of Aristotle and Galileo, and Descartes third law preceded Newton's third law.

**Aristotle:** The first part of Newton's first law of motion is nothing but Aristotle assertion regarding moving bodies i.e. bodies have natural tendency to remain at rest. It is set in motion when impressed force acts on it. *Every body perseveres in its state of rest, unless it is compelled to change that state by forces impressed thereon.*
Aristotle's assertion was contradicted by Galileo ( however for hypothetical system) and now abandoned. **Newton:** The second part of Newton's first law of motion is just Galileo's law of Inertia which is established for hypothetical medium devoid of resistive forces.

*Every body perseveres in its state of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon.*

In second law of motion, Newton gave equation to calculate the force when velocity of body varies i.e. F = (v-u). Thus Galileo's law of inertia which was used to contradict Aristotle's assertion plays a significant role. **Descartes:** Newton's third law of motion was definitely anticipated by Descartes about 42 years ago , as we understand the same daily life. It can be further expanded to third case not directly discussed by Descartes, but it is obvious deduction from the law. Newton had given very compact definition of third law of motion. Newton had studied Descartes work closely and with help of prism Newton contradicted Rene Descartes' theory of light.

## VIII. CONCLUSIONS

The experimental findings and logical theoretical deductions always have upper hand over established laws in many cases. The laws of science are relative not absolute, like laws of arts. The laws of arts change automatically in many cases, whereas laws of science need permission from peers.

(i) Newton's corpuscular theory predicts the speed of light in water, must be greater than the speed of light in vacuum. It is clear experimental contradiction. Thus it was replaced by Huygens's theory of light which predicts light travels in form of waves and explains other phenomena as well.
(ii) Speed of sound in air: In the Principia Newton stated that the sound in air propagates isothermally. Thus ,
$v= (P/D)^{1/2} = 280$ m/s

v is speed of sound , P is atmospheric pressure , D is density of atmosphere . Newton assumed that speed of sound in air is under isothermal conditions. Thus value of speed of sound turns out to be 280 m/s, whereas actual value is 331m/s i.e. underestimation of 15%.

Whereas French mathematician Pierre-Simon Laplace saw the flaw in Newton's perceptions and ultimately corrected Newton's formula that sound propagates adiabatically i.e.

$$v = (\gamma\ P/D)^{1/2} =330\text{m/s}$$

In this case actual speed of sound is calculated i.e. 330m/s

Thus Laplace's equation was found valid and used now and Newton's equation is abandoned. There are more examples regarding it in science.

Finally we conclude that Newton's first law of motion is combined effect of Aristotle assertion and Galileo's law of inertia. In second law of motion, Newton gave equation of force when velocity varies i.e. F =(v-u). However now equation for second of motion is used as F =ma which was given by Euler in 1775.

Newton's third law of motion was definitely preceded in 1644 by Descartes third law of motion, in elaborated way. Further Newton's third law of motion has been generalized ( equally applicable for Descartes law ) that it does not take in account the inherent nature and characteristics of bodies like wooden ball, sponge ball , chewing gum ball and irregular shaped iron body etc.

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In Law of gravitation Newton believed that planets revolve around sun in definite orbits. It is deduction in Kelper's law published in 1609, on the basis of Tycho Brahe's lifelong observations. The gravitation force varies as inverse square of distance ( $r^{-2}$) of two bodies, was contested by Robert Hooke. Further Newton has himself acknowledged the contribution of Christophor Wren , Robert Hooke and Edmund Halley regarding it in the Principia. However in the various propositions I-XIII in Book III, Newton did not mention at all that force of attraction between various bodies is product of masses. The value of G ( $F = GmM/r^2$) was measured by Cavendish in 1798 . However Newton did not give any equation for the laws in the Principia. Thus Newton critically analyzed the various perceptions from existing literature to formulate the various laws which are exceptionally useful since centuries.

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